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STABILIZATION OF AN AXISYMMETRIC LIQUID BRIDGE BY VISCOUS FLOW

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Abstract—An axial flow of an immiscible liquid past a nearly cylindrical liquid bridge of similar density influences the stability of the bridge. Laminar flow along a cylindrical interface can generate a differential pressure gradient across the interface that is equivalent to a steady axial gravitational acceleration. Ideally, this flow can be used to cancel the pressure gradient due to the density imbalance, allowing a longer cylindrical interface than would be statistically possible. Experimental results that show this effect and that are in qualitative agreement with predictions are presented.

Key Word: stability, annular, laminar, capillary, cylinder, liquid, bridge

INTRODUCTION

A liquid bridge forms when a drop of liquid atop the end of a cylindrical rod, held vertically, is brought into contact with the end of a second cylindrical rod positioned above it. In our apparatus the rods are coaxial and the ends are wet with the bridge liquid; the contact-line remains fixed at the edge of the rod. The bridge is immersed in a second immiscible liquid with nearly the same density. In contrast to the classical Plateau chamber where the outer container is closed, the outer chamber in our apparatus is a long pipe with the entrance connected to a constant-pressure head tank. Thus, the outer liquid is part of a continuous flow loop and the inner liquid is held between two long cylindrical rods which are centred in the cylindrical pipe.

In the absence of fluid motion and for a perfect density match, the liquid/liquid interface can take the shape of a circular cylinder. It is a classical result that such a cylindrical interface is unstable when its length exceeds its circumference (capillary instability), often called the Rayleigh limit (Plateau 1873; Rayleigh 1879). When the liquid/liquid densities are not perfectly matched, the equilibrium shapes become unstable at lengths shorter than the Rayleigh limit; gravity is destabilizing (e.g. Perales & Meseguer 1991). Various other modifications of this configuration for a finite bridge, including uniform rotation, have been studied and such modifications are similarly destabilizing (e.g. Michael 1981; Ungar & Brown 1982). On the other hand, recent theoretical studies of the stability of infinite cylinders show that for certain axial flow profiles the capillary instability can be stabilized (Russo & Steen 1989). A similar effect of "shear stabilization" has been observed in experimental studies of core-annular flows in the laminar regime (e.g. Hu *et al.* 1990).

In this paper we present observations of bridges in the presence of a surrounding pipe flow that are longer than those which occur in the absence of flow. We are concerned with bridges that are axisymmetric and nearly cylindrical (i.e. nearly neutrally buoyant). The principal observations are explained in terms of the axial pressure gradient of the surrounding flow which can oppose a small hydrostatic gradient. Several other observations are recorded that are not explained by the simple theory. These will be explored elsewhere.

STATIC SHAPES AND STABILITY

In the absence of any fluid motions, the governing equations reduce to the normal stress balance across the interface, a form of the Young–Laplace equation. In a cylindrical system with the z-coordinate along the rod axis, the dimensionless equation can be written,

$$[p] = 2\kappa + \mathrm{B}z. \qquad [1]$$

Here, $[p]$ represents the pressure difference across the interface, in this case a constant equal to the static pressure difference at $z = 0$; κ is the mean curvature of the interface and B (Bond number) represents the ratio of gravitational to surface forces, $B \equiv gr^2\Delta\rho/\sigma$. Scales of r and σ/r for lengths and pressure, respectively, have been used in [1], where r is the rod radius and σ the surface tension. In the definition of B, the density difference of the incompressible liquids $\Delta\rho$ and the gravitational acceleration g also appear. In experiments, the static pressure difference is determined by the volume of the inner bridge fluid, a control parameter.

Equation [1] governs the equilibrium shapes of the interface. Axisymmetric shapes satisfying the boundary conditions have been obtained for $B \neq 0$ in many studies (e.g. Perales & Meseguer 1991; Coriell *et al.* 1977; Sanz & Martinez 1983). Solutions in the special case $B = 0$ correspond to surfaces of revolution of pieces of elliptic integrals and are classical (Howe 1887; Gillette & Dyson 1979). The cylindrical interface is a member of this family of solutions. The stability limit for the cylindrical interface, also classical, is obtainable alternatively by a static energy stability criterion (Plateau 1873) or by a dynamic linear stability analysis (Rayleigh 1879). The separation of the rods relative to their radius is L and the stability limit at $B = 0$ is $L_c = 2\pi$. Figure 1 shows the influence of B on L_c (Sanz & Martinez 1983). Note the sensitivity of L_c (B); a change of 0.01 in B can give an 8% change in L_c near $B = 0$.

In the experiment, the length L is quasistatically increased with the volume maintained at that which would be contained by a cylindrical interface until instability occurs. Figure 2 shows the bridge with a centre rod in place. It serves to influence axial pressure gradients in the bridge when there is flow. Note that this rod does not affect L_c under static conditions.

FLOW EFFECTS

The flow in the finite bridge (figure 2) is approximated by an axially infinite flow justified in part by the small aspect ratio of the bridge annulus. Specifically, we note that a modified pipe flow satisfying (i) no slip at the centre connecting rod and at the outer tube wall, (ii) continuity of velocity and shear across the liquid/liquid interface, (iii) no net flow through the annular cross-section of the bridge liquid, and (iv) a prescribed net flow Q through the outer annulus is an exact solution of the Navier–Stokes equations provided the surface tension is large compared to viscous forces (small capillary number). This solution has the property that although the axial

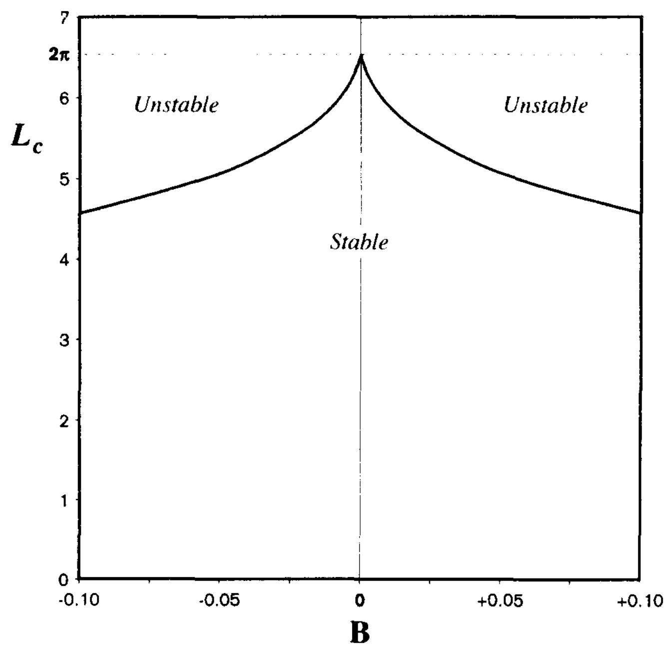
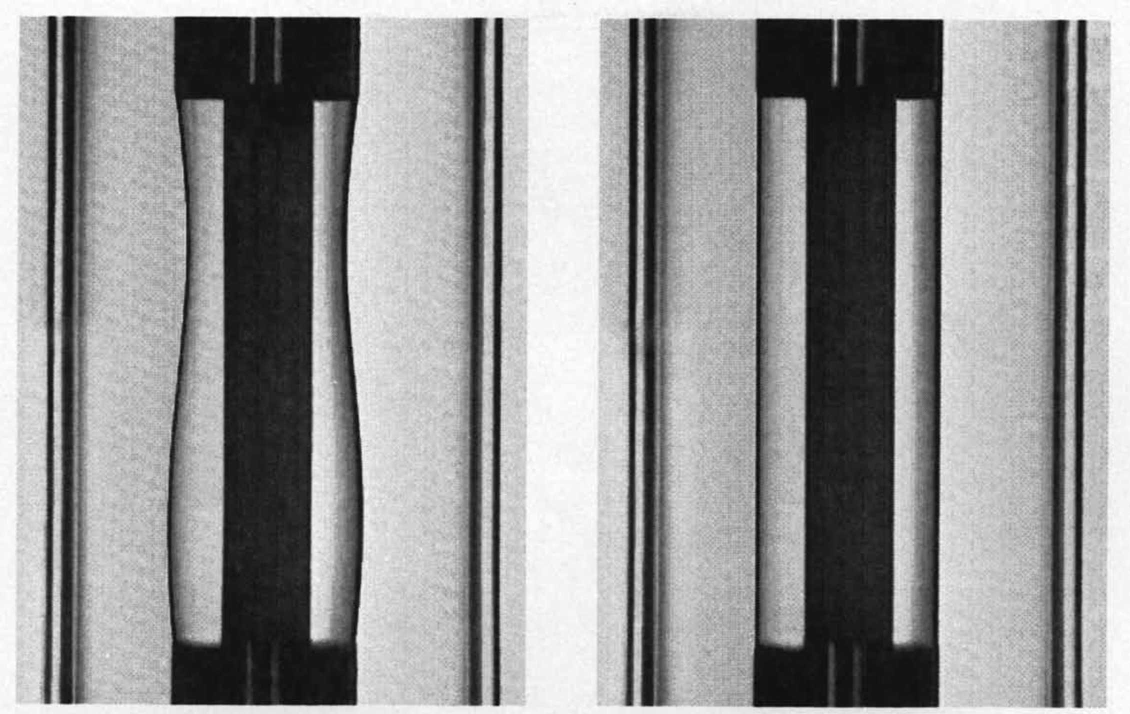


Figure 1. Stability limits for static liquid bridges from theory (volume constrained to that of a cylinder of equal length).



(a) (b)

Figure 2. Interface shape of stable bridges from video: (a) no flow (C = 0); (b) flow (C = 0.006). Centre rod radius: $r_i = 6.4$ mm; contact line radius: $r = 14.1$ mm; outer tube radius: $r_o = 34.0$ mm; $L = 5.65$; $B = 0.010 \pm 0.001$.

pressure gradient in each phase is constant, these constants differ. It is straightforward to derive the expression for this difference (dimensional)

$$\frac{\mathrm{d}}{\mathrm{d}z}[p] = -A\frac{\eta_o Q}{r^4}, \qquad [2]$$

where $A\,(r_i/r,\ r_o/r,\ \eta_{io})$ is a constant dependent on the centre rod radius r_i, the outer tube radius r_o and the viscosity ratio $\eta_{io} \equiv \eta_i/\eta_o$. In essence the outer flow drives a return flow in the liquid bridge, forcing an axial pressure gradient in the interior opposite to that of the outer flow.

The influence of flow is accounted for to a first approximation as a modification of the static balance, [1]. Using the scales introduced for [1], [2] can be rewritten in non-dimensional form:

$$\frac{\mathrm{d}}{\mathrm{d}z}[p] = -\mathrm{C}, \qquad [3]$$

where the capillary number $\mathrm{C} \equiv A\eta_o Q/\sigma r^2$. Including this effect in [1] gives

$$[p] = 2\kappa + (\mathrm{B} - \mathrm{C})z, \qquad [4]$$

which assumes that the flow influences the interface shape only through the axial pressure gradient in [3]. The coefficient B − C may be viewed as an effective Bond number. In view of figure 1, maximum L_c might be expected for flows that make a cylindrical interface possible (B = C).

EXPERIMENTAL RESULTS

Figure 2(a) shows the bridge configuration without flow. The bridge liquid is a mixture of two fluorotoluene isomers, whose densities straddle that of the purified water used in the outer flow. The bridge is slightly heavy, corresponding to a small positive B. B values are not measured independently but are estimated using the measured L_c and theory (as in figure 1). Effects that appear to influence B include molecular diffusion and ambient temperature variations. The error

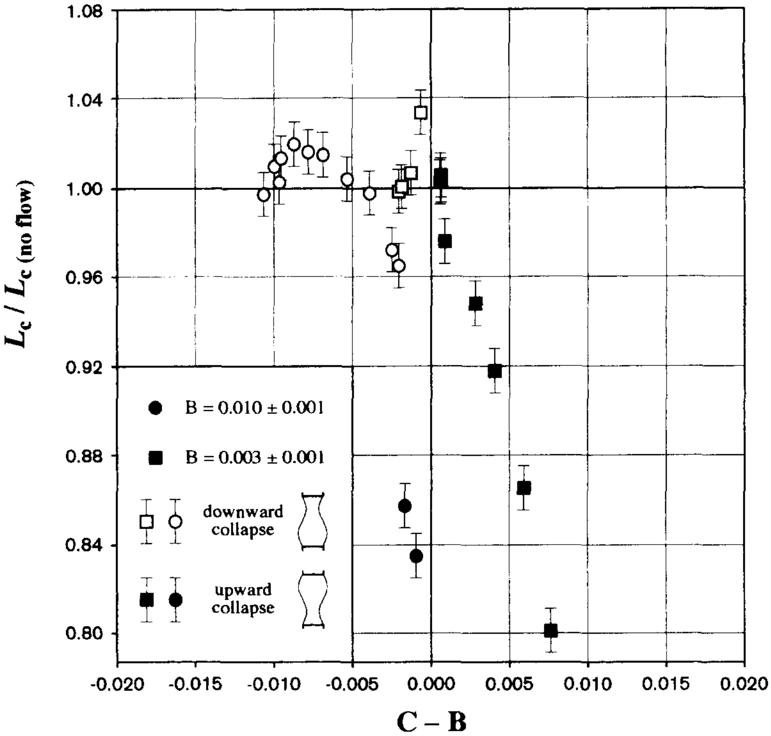


Figure 3. Normalized stability limit vs effective B for two experimental runs. $r_i/r = 0.643$, $r_o/r = 2.403$, $\eta_{io} = 0.89$.

indicated for B reflects such cumulative effects as measured by a shift in L_c over the duration of a run (3 h). B is considered fixed in interpreting the flow experiments.

A flow from bottom to top is imposed and as the flow rate increases shape changes smoothly from figure 2(a) to that shown in figure 2(b). This agrees qualitatively with the predictions of [4] for increasing C. Note, however, that the interface appears cylindrical at flow rates below C = B. Figure 3 shows a quantitative comparison. In figure 3, the stability limit with flow normalized by that measured with no flow is plotted against C – B. These data are typical, in that they show (i) measurably longer bridges with flow than without, (ii) a flip in the collapse character (downward to upward bulge) near C = B and (iii) significantly shorter bridges at higher flow rates. Several features are not explained by [4], however. For B = 0.01, the greatest stabilization occurs for C < B. Furthermore, the data for this case are clearly not symmetric about their maximum. These discrepancies with [4] are attributed to the approximations embodied in that equation, which neglects, among others, end effects, inertia, large interface distortions and interface contamination. We will not explore these secondary effects further here.

CONCLUSIONS

By using the differential pressure created by laminar flow about a liquid bridge, stable bridges of length longer than possible for the static case have been obtained. The flow can suppress deviations from a cylindrical interface and, as expected from static theory, longer stable bridges are possible for such interfaces. This effect is most pronounced for small gravitational imbalances, where the stability limit is most sensitive to the flow rate. The model introduced qualitatively accounts for the principal features but leaves some secondary effects unexplained.

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